International Meeting on Commutative Monoids 2025
Tenerife, Canary Islands, Spain

Aula 5-6, Sección de Matemáticas, Universidad de La Laguna

The *International Meeting on Commutative Monoids 2025* will be held at Universidad de La Laguna (Tenerife, Spain) from 14th to 16th July 2025. The recent results on commutative monoids will be discussed.

	Monday 14	Tuesday 15	Wednesday 16
	Chairman: García Marco	*Chairwoman: Márquez Corbella*	*Chairman: Santana Sánchez*
09:30 -10:00	Registration and Opening	M. Branco	S. Eliahou
10:10 -10:40	H. Srinivasan	M. Delgado	A. Fidalgo
10:50 -11:30	Coffee	Coffee	Coffee
11:30 -12:00	J. Moyano	J. I. García	D. Llena
12:10 -12:40	M. Revuelta	A. Vigneron	F. de la Rosa
12:50 -13:20	M. González	R. Tapia	W. Hernández
13:30 - 16:30	Free time	Free time	Free time
16:30 - 18:30	Working groups	Visit city center (17:00-18:30)	Working groups
	Social dinner (20:00)		

Each talk will consist of 30 minutes of presentation and 5 minutes of questions and discussion.

During Monday and Wednesday afternoons, working groups will develop.

On Monday 14th the social dinner will be held at the Patio Canario de La Laguna, calle Manuel de Ossuna, 8, La Laguna.

On Tuesday 15th, from 17:00 to 18:30 there will be a guided tour of the city centre, organised by the La Laguna City Council (free).

Organised by the GASIULL Research Group (Universidad de La Laguna).

Sponsored by the project *Monoids and related semigroups (ProyExcel 00868).* Project funded in the 2021 call for Excellence Project Grants (PAIDI 2020).

Scientific committee:
Shalom Eliahou (Université du Littoral Côte d'Opale)
José Carlos Rosales (Universidad de Granada)
Alberto Vigneron-Tenorio (Universidad de Cádiz)

Organizing committee:
Evelia R. García Barroso (Universidad de La Laguna)
Ignacio García Marco (Universidad de La Laguna)
Irene Márquez Corbella (Universidad de La Laguna)

Luis J. Santana Sánchez (Universidad de La Laguna)

Official language: The meeting has no official language.

Webpage: https://imcm2025.uca.es/

Institutional Partners: Sección de Matemáticas, Instituto de Matemáticas y Aplicaciones (IMAULL) and Área of Álgebra of Universidad de La Laguna. La Laguna City Council.

Conferencias invitadas

The covariety of bracelet numerical semigroups
Manuel B. Branco (Universidade de Évora, Portugal)

Given a set of positive integers $\{n_1, \ldots, n_p\}$, we say that a numerical semigroup S is a $\{n_1, \ldots, n_p\}$-bracelet if $a+b+\{n_1, \ldots, n_p\} \subseteq S$ for every $a, b \in S \setminus \{0\}$. In this talk we study the set of $\{n_1, \ldots, n_p\}$-bracelet numerical semigroups with fixed Frobenius number, denoted by $\mathcal{B}\big(\{n_1, \ldots, n_p\}, F\big)$. We will prove that $\mathcal{B}\big(\{n_1, \ldots, n_p\}, F\big)$ is a covariety and we will give algorithms for computing all elements in $\mathcal{B}\big(\{n_1, \ldots, n_p\}, F\big)$ or all elements in $\mathcal{B}\big(\{n_1, \ldots, n_p\}, F\big)$ with fixed genus.

Joint work with J. C. Rosales and Ilvécio Ramos.

Counting and verifying properties on numerical semigroups up to high genus
Manuel Delgado (Universidade do Porto, Portugal)

I plan to recall a well-known process of counting numerical semigroups by exploring the classical numerical semigroups tree. Through the insertion of some existing theory, one can count up to a higher genus without increasing the computational cost. Furthermore, the method gets better as the genus increases.

Notice that the process of counting is easily adapted to verify properties: one verifies the property at each explored node. In this case, one can incorporate additional existing theoretical results, some of which lead to relatively efficient computations. With S. Eliahou and J. Fromentin we obtained the most relevant result in this direction published up to now: there is no counter-example to Wilf's conjecture among the many numerical semigroups with genus not greater than 100.

I plan to give an overview of some work done on this topic.

On the bases of the toric ideal of a numerical semigroup
Fernando de la Rosa (Universidad de La Laguna, Spain)

Toric ideals can be described by means of several generating sets, such as the Graver basis, the (reduced) Gröbner basis with respect to a monomial order and Markov bases. In this talk, we focus on toric ideals arising from numerical semigroups and present new upper bounds on the degrees of the binomials in these bases. These bounds improve existing bounds for general toric ideals.

This is a report on an ongoing joint work with Ignacio García Marco.

On the counting function of numerical semigroups of multiplicity 5
Shalom Eliahou (Université du Littoral Côte d'Opale, France)

Let $n(g, m)$ denote the number of numerical semigroups of genus g and multiplicity m. Kaplan proved in 2012 that, for all m, the function $n(g, m)$ is given asymptotically by a quasi-polynomial in g of degree $m - 2$, and conjectured the inequality $n(g, m) \geq n(g-1, m)$ for all g, as a refinement of a 2006 conjecture by Bras-Amorós on the growth of the numerical semigroups tree. Kaplan's conjecture is open for $m \geq 6$. In multiplicity $m = 5$, García-Sánchez, Marín-Aragón and Robles-Pérez gave a long explicit formula for $n(g, 5)$ in a 2018 arXiv preprint, from which they proved the conjectured growth $n(g, 5) \geq n(g - 1, 5)$ for all g. These results were obtained with a computer-intensive method to count integer points in polytopes. In this talk, with a different method, we provide a new proof of the inequality $n(g, 5) \geq n(g - 1, 5)$, together with a new shorter formula for $n(g, 5)$, as an explicit quasi-polynomial of period 30 and dominant term $g^3/135$. Our method has the potential to tackle cases of higher multiplicity.

A numerical semigroup problem arising from decoding algebraic-geometry codes
Adrián Fidalgo Díaz (Universidad de Valladolid, Spain)

One-point algebraic-geometry codes form a well-studied class of error-correcting codes. One of their main virtues is that they can approach the Singleton bound (assuming the genus of the curve is low) while potentially having a length greater than the size of the field (a limitation that Reed-Solomon codes suffer from). This construction is based on evaluating algebraic functions whose pole set is restricted to a single point and a given order. The set of possible pole orders at this point forms a numerical semigroup, often referred to as the Weierstrass semigroup.

In this talk, we will state a problem defined only in terms of the Weierstrass semigroup which has implications for decoding one-point algebraic-geometry codes. We will solve this problem for some families of numerical semigroups arising from curves, while placing the problem in the context of its applications.

On ideals of affine semigroups and affine semigroups with maximal embedding dimension
Juan Ignacio García García (Universidad de Cádiz, Spain)

Let $S \subseteq \mathbb{N}^p$ be a semigroup, any $P \subseteq S$ is an ideal of S if $P + S \subseteq P$, and an $I(S)$-semigroup is the affine semigroup $P \cup \{0\}$, with P an ideal of S. We characterise the $I(S)$-semigroups and the ones that also are $\mathcal{C}$-semigroups. Moreover, some algorithms are provided to compute all the $I(S)$-semigroups satisfying some properties. From a family of ideals of S, we introduce the affine semigroups with maximal embedding dimension, characterising them and describing some families.

On the Betti numbers of Kunz-Waldi semigroups

Mario González Sánchez (Universidad de Valladolid, Spain)

Given two coprime numbers $p < q$, KW semigroups, introduced by Kunz and Waldi, contain p, q and are contained in $\langle p, q, r \rangle$ where $2r = p, q, p + q$ whichever is even. Kunz and Waldi proved that all KW semigroups of embedding dimension $n \geq 4$ have Cohen-Macaulay type $n - 1$ and first Betti number $\binom{n}{2}$.
In this talk, we give a characterization of the KW semigroups whose defining ideal is generated by the 2×2 minors of a matrix. In addition, we identify all KW semigroups that lie on the interior of the same face of the Kunz cone $\mathcal{C}_p$ as a KW semigroup with determinantal defining ideal. Thus, we provide an explicit formula for the Betti numbers of all such semigroup rings: $\beta_0 = 1$, and $\beta_i = i\binom{n}{i+1}$ for $i = 1, \ldots, n - 1$.
This talk is based on a joint work with S. Singh and H. Srinivasan.

Some properties on GSI semigroups

William Giovanni Hernández Yanes (Universidad de La Laguna, Spain)

We present the family of GSI semigroups introduced in [1] and compare it with other notable families of semigroups. Moreover we give some properties of them, in particular the description of their pseudo-Frobenius numbers and the set of special gaps.

These results are part of my Master's thesis.

References

[1] E.R. García Barroso-J.I. García-García-A. Vigneron-Tenorio, Generalized strongly increasing semigroups. Mathematics 2021, 9, 1370. https://doi.org/10.3390/ math9121370.

QMED-semigroups (some advances)

David Llena Carrasco (Universidad de Almería, Spain)

QMED-semigroups have been introduced in the recent paper *Quasi maximal embedding dimension semigroups*. Ricerche di Matematica. (2025) 74, 731-749. We present some news about such numerical semigroups.

Unveiling a monoid structure in score sheets

Julio José Moyano Fernández (Universitat Jaume I, Castellón, Spain)

The set M_n of ordered score sheets of a round-robin football tournament played between n teams together with the pointwise addition has the structure of an affine monoid. We present the study of the most important invariants of this monoid, namely the Hilbert basis, the multiplicity, the Hilbert series and the Hilbert function, which has been done by using Normaliz. In addition, we introduce the submonoids R_n resp. C_n of M_n for which the order is preserved after the leader team is disqualified, resp. all principal submatrices preserve the given ordering. Surprisingly, C_n turns out to be Gorenstein for $n > 2$.

This is a joint project with Bogdan Ichim.

Sum-Free Sets and Ideal Graphs in Numerical Semigroups
María Revuelta Marchena (Universidad de Sevilla, Spain)

Let S be a numerical semigroup and $I \subsetneq S$ a proper ideal. We define the additive graph

$$G_I(S) := \big(V(G_I), E(G_I)\big),$$

where the vertex set is

$$V(G_I) = (S \setminus I) \setminus \{0\},$$

and there is an edge $\{x, y\} \in E(G_I)$ if and only if $x + y \in I$.
In this work we formulate conjectures relating the following invariants:

- The clique number $\omega\big(G_I(S)\big)$ of the graph $G_I(S)$.
- The independence number $\alpha\big(G_I(S)\big)$ of the graph $G_I(S)$.
- The maximum size of a sum-free subset in $S \setminus I$, namely

$$\max\big\{|A| : A \subseteq S \setminus I,\ \forall\, x, y \in A,\ x + y \notin A\big\}.$$

We conjecture, in particular, that

$$\omega\big(G_I(S)\big) \ = \ \max\big\{|A| : A \subseteq S \setminus I,\ A \text{ is sum-free}\big\},$$

and that

$$\alpha\big(G_I(S)\big) \ \leq \ \omega\big(G_I(S)\big) \ + \ 1.$$

These conjectures are illustrated in symmetric, pseudo-symmetric, and irreducible numerical semigroups, accompanied by computational analysis for small genera. Finally, we discuss open research directions, including proving the aforementioned conjectures, studying additional graph parameters (diameter, girth, planarity), and systematically generating instances via algorithmic tools.

Toric Gluing and Toric Splitting
Hema Srinivasan (University of Missouri, USA)

The notion of gluing two commutative monoids is closely related to the notion of splitting of toric ideals. If I_1 and I_2 are the toric ideals associated to the commutative monoids A_1 and A_2 respectively, then the ideal J of the gluing of A_1 and A_2 is given by $J = I_1 + I_2 + (f)$ where f is the gluing binomial. When f is of standard degree 1, then we can simply eliminate a variable and have the new toric ideal as $I_1 + I_2$. This is the splitting of the toric ideal J as the sum of two toric ideals whose minimal resolution is given by the tensor product of the resolutions of the two parts. We will discuss our on going projects with Philippe Gimenez on this topic. In particular we will show how to explicitly glue or split in the homogenous case as well as some non homogenous cases as well.

Current Developments and Unresolved Questions on Positioned $\mathcal{C}$-Semigroups

Raquel Tapia Ramos (Universidad de Cádiz, Spain)

Let $\mathcal{C}$ be a positive integer cone. A $\mathcal{C}$-semigroup $S \subseteq \mathcal{C}$ is a cofinite affine semigroup. Fixed $k \in \mathcal{C}$, we say that a $\mathcal{C}$-semigroup S is k-positioned if for every gap h, the element $k - h$ belongs to S. Positioned $\mathcal{C}$-semigroups are born as a natural generalization of symmetric numerical semigroups, where the Frobenius number is replaced by $k \in \mathcal{C}$.
In this talk, we will focus on a special subfamily called primary positioned $\mathcal{C}$-semigroups, which can be graphically classified via a rooted tree. In particular, we will analyse in detail primary positioned $\mathbb{N}^p$-semigroups, emphasizing the differences that arise when $\mathcal{C} \neq \mathbb{N}^p$. Several examples and algorithmic procedures will be presented.
It is a joint work with Carmelo Cisto. The author is partially supported by the project ProyExcel_00868 (Proyecto financiado en la convocatoria 2021 de Ayudas a Proyectos de Excelencia, en régimen de concurrencia competitiva, destinadas a entidades calificadas como Agentes del Sistema Andaluz del Conocimiento, en el ámbito del Plan Andaluz de Investigación, Desarrollo e Innovación (PAIDI 2020). Consejería de Universidad, Investigación e Innovación de la Junta de Andalucía.)

Generalizing $\mathcal{A}$-semigroups

Alberto Vigneron Tenorio (Universidad de Cádiz, Spain)

Consider $\mathcal{C} \subseteq \mathbb{N}^p$ a non-negative integer cone. An affine semigroup is a $\mathcal{C}$-semigroup if the minimal integer cone containing S is $\mathcal{C}$, and $\mathcal{C} \setminus S$ is a finite set. In case $p = 1$, S is a numerical semigroup.
For a numerical semigroup $S \subseteq \mathbb{N}$ with Frobenius number f, S is called an $\mathcal{A}$-semigroup if, for any $x \in \mathbb{N}$ where $x < f$, the condition $\{x, x+1\} \not\subset S$ holds (see [1]). The elements of S that are smaller than its Frobenius number are referred to as *small elements*.
In this talk, we generalize the concept of $\mathcal{A}$-semigroup to $\mathcal{C}$-semigroup (i.e. for $p > 1$) and explore some of their properties. These results are part of an ongoing joint work with J.C. Rosales (Universidad de Granada) and R. Tapia-Ramos (Universidad de Cádiz).

References

[1] Rosales, J. C.; Branco, M. B.; Traesel, M. A., Numerical semigroups without consecutive small elements. Internat. J. Algebra Comput. 33 (2023), no. 1, 67–85.

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